MTH30 Test 1 Preparation Chapter 1 : 1.4, 1.5, 1.6, 1.8

True - False questions, p. 94.

$\boxed{N7}$ $\boxed{\text{False}}$ because both limits $\lim\limits_{x \to 4} \frac{2x}{x-4}$ and $\lim\limits_{x \to 4} \frac{3}{x-4}$ do not exist.

$\boxed{N11}$ $\boxed{\text{False}}$, counter example:

let $f(x) = x^2 - 5x$, $g(x) = x - 5$. Then $\lim\limits_{x \to 5} f(x) = 0$, $\lim\limits_{x \to 5} g(x) = 0$

but $\lim\limits_{x \to 5} \frac{f(x)}{g(x)} = \lim\limits_{x \to 5} \frac{x^2 - 5x}{x - 5} = \lim\limits_{x \to 5} \frac{x\cancel{(x-5)}}{\cancel{x-5}} = \lim\limits_{x \to 5} x = 5$.

$\boxed{N15}$ $\boxed{\text{True}}$ (using Direct Distribution Property, and polynomial function's domain is $\mathbb{R}$)

$\boxed{N17}$ True (if it's a vertical assymptote, that the function has $+\infty$ or $-\infty$ as one of its limits)

Exercises, p. 96

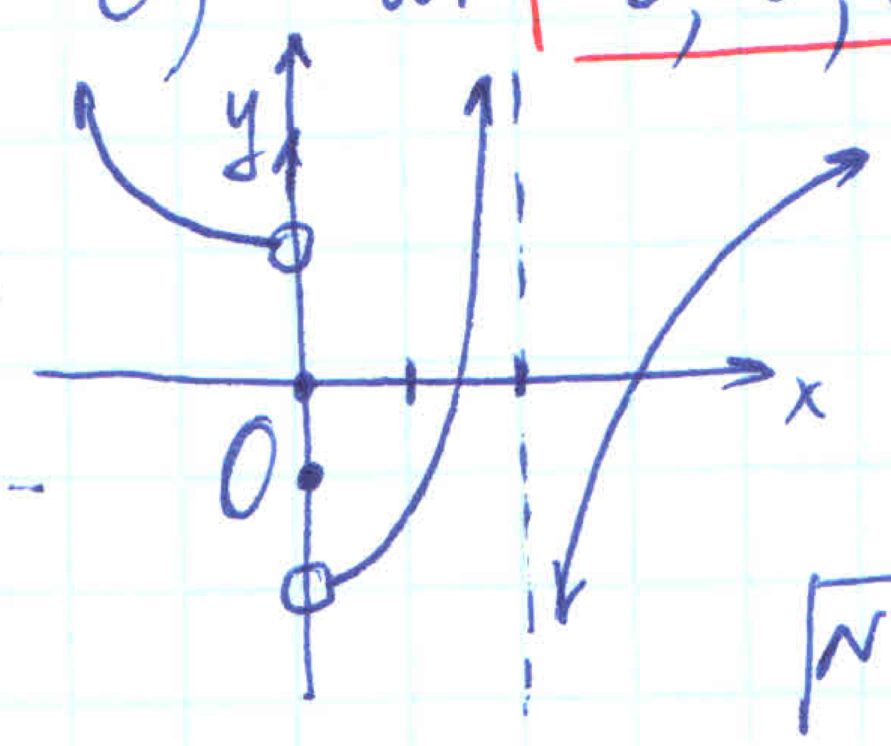
$\boxed{N23}$ (a) (i) $\boxed{3}$ (ii) $\boxed{0}$ (iii) $\boxed{\text{does not exist}}$

(iv) $\boxed{2}$ (v) $\boxed{+\infty}$ (vi) $\boxed{-\infty}$

b) $\boxed{x = 0}$, $\boxed{x = 2}$

c) at $\boxed{-3, 0, 2, 4.}$

$\boxed{N24}$



$\boxed{N25}$ $\lim\limits_{x \to 0} (\cos(x + \sin x)) =$

$= \cos(0 + \sin 0) = \cos 0 = \boxed{1}$

$\boxed{N26}$ $\lim\limits_{x \to 3} \frac{x^2 - 9}{x^2 + 2x - 3} = \frac{3^2 - 9}{3^2 + 2 \cdot 3 - 3} = \frac{0}{12} = \boxed{0}$

$\boxed{N27}$ $\lim\limits_{x \to -3} \frac{x^2 - 9}{x^2 + 2x - 3} = \lim\limits_{x \to -3} \frac{\cancel{(x+3)}(x-3)}{\cancel{(x+3)}(x-1)} = \frac{(-3) - 3}{(-3) - 1} = \frac{-6}{-4} = \boxed{\frac{3}{2}}$

$\boxed{N28}$ $\lim\limits_{x\to 1^+} \dfrac{x^2-9}{x^2+2x-3} = \lim\limits_{x\to 1^+} \cancel{\dfrac{(x+3)(x-3)}{(x+3)(x-1)}} \quad \dfrac{1-9}{\cdot} = \boxed{-\infty}$

not helping (stil getting 0 in the denominator) and is very very small (close to 0)

$\begin{cases} x^2-9<0 \text{ as } x\to 1^+ \\ x^2+2x-3>0 \text{ as } x\to 1^+ \end{cases}$

Hence $\dfrac{x^2-9}{x^2+2x-3} \to -\infty$ as $x\to 1^+$

$\boxed{N29}$ $\lim\limits_{h\to 0} \dfrac{(h-1)^3+1}{h} = \lim\limits_{h\to 0} \dfrac{h^3-3h^2+3h\cancel{-1}\cancel{+1}}{h} = \lim\limits_{h\to 0} \dfrac{\cancel{h}(h^2-3h+3)}{\cancel{h}} =$

$= \lim\limits_{h\to 0}(h^2-3h+3) = \boxed{3}$

$\boxed{N30}$ $\lim\limits_{t\to 2} \dfrac{t^2-4}{t^3-8} = \lim\limits_{t\to 2} \dfrac{\cancel{(t-2)}(t+2)}{\cancel{(t-2)}(t^2+2t+4)} = \lim\limits_{t\to 2} \dfrac{t+2}{t^2+2t+4} =$

$$\begin{array}{r} t^2+2t+4 \\ t-2\,\overline{)\,t^3-8} \\ -\;t^3-2t^2 \\ \hline 2t^2-8 \\ -\;2t^2-4t \\ \hline 4t-8 \\ -\;4t-8 \\ \hline 0 \end{array}$$

$= \dfrac{2+2}{2^2+2\cdot 2+4} = \dfrac{4}{12} = \boxed{\dfrac{1}{3}}$

$\boxed{N31}$ $\lim\limits_{r\to 9} \dfrac{\sqrt{r}}{(r-9)^4} = \boxed{\infty}$

$\sqrt{r} \to 3$ as $r\to 9$; $(r-9)^4 > 0$ and is very very small, (close to 0) as $r\to 9$

$\boxed{N32}$ $\lim\limits_{v\to 4^+} \dfrac{4-v}{|4-v|} = \boxed{-1}$

$4-v = -|4-v|$ for all $v \geq 4$

$\boxed{N33}$ $\lim\limits_{u\to 1} \dfrac{u^4-1}{u^3+5u^2-6u} = \lim\limits_{u\to 1} \dfrac{(u^2-1)(u^2+1)}{u(u^2+5u-6)} = \lim\limits_{u\to 1} \dfrac{\cancel{(u-1)}(u+1)(u^2+1)}{u\cancel{(u-1)}(u+6)}$

$= \lim\limits_{u\to 1} \dfrac{(u+1)(u^2+1)}{u(u+6)} = \dfrac{2\cdot 2}{1\cdot 7} = \boxed{\dfrac{4}{7}}$

$\boxed{N34}$ $\lim\limits_{x\to 3} \dfrac{\sqrt{x+6}-x}{x^3-3x^2} = \lim\limits_{x\to 3} \dfrac{(\sqrt{x+6}-x)(\sqrt{x+6}+x)}{x^2(x-3)} =$

$= \lim\limits_{x\to 3} \dfrac{x+6-x^2}{x^2(x-3)} = \lim\limits_{x\to 3} \dfrac{-(x^2-x-6)}{x^2(x-3)} = \lim\limits_{x\to 3} \dfrac{-\cancel{(x-3)}(x-2)}{x^2\cancel{(x-3)}} =$

$= \lim\limits_{x\to 3} \dfrac{-(x-2)}{x^2} = \boxed{\dfrac{-1}{9}}$

$\boxed{N35}$ $\lim\limits_{s\to 16} \dfrac{4-\sqrt{s}}{s-16} = \lim\limits_{s\to 16} \dfrac{4-\sqrt{s}}{s-16} \cdot \dfrac{4+\sqrt{s}}{4+\sqrt{s}} = \lim\limits_{s\to 16} \dfrac{16-s}{(s-16)(4+\sqrt{s})} =$

$= \lim\limits_{s\to 16} \dfrac{-1}{4+\sqrt{s}} = \dfrac{-1}{4+4} = \boxed{-\dfrac{1}{8}}$

$\boxed{N36}$ $\lim\limits_{v\to 2} \dfrac{v^2+2v-8}{v^4-16} = \lim\limits_{v\to 2} \dfrac{(v+4)(v-2)}{(v^2-4)(v^2+4)} = \lim\limits_{v\to 2} \dfrac{(v+4)\cancel{(v-2)}}{\cancel{(v-2)}(v+2)(v^2+4)} =$

$= \lim\limits_{v\to 2} \dfrac{v+4}{(v+2)(v^2+4)} = \dfrac{2+4}{4\cdot 8} = \dfrac{6}{32} = \boxed{\dfrac{3}{16}}$

$\boxed{N37}$ $\lim\limits_{x\to 0} \dfrac{1-\sqrt{1-x^2}}{x} = \lim\limits_{x\to 0} \dfrac{1-\sqrt{1-x^2}}{x} \cdot \dfrac{1+\sqrt{1-x^2}}{1+\sqrt{1-x^2}} =$

$= \lim\limits_{x\to 0} \dfrac{1-(1-x^2)}{x(1+\sqrt{1-x^2})} = \lim\limits_{x\to 0} \dfrac{\overset{x}{\cancel{x^2}}}{\cancel{x}(1+\sqrt{1-x^2})} = \lim\limits_{x\to 0} \dfrac{x}{1+\sqrt{1-x^2}} = \dfrac{0}{2} = \boxed{0}$

$\boxed{N38}$ $\lim\limits_{x\to 1} \left(\dfrac{1}{x-1} + \dfrac{1}{x^2-3x+2}\right) = \lim\limits_{x\to 1} \left(\dfrac{\overset{*(x-2)}{1}}{\underset{*(x-2)}{x-1}} + \dfrac{1}{(x-1)(x-2)}\right) =$

$= \lim\limits_{x\to 1} \dfrac{x-2+1}{(x-1)(x-2)} = \lim\limits_{x\to 1} \dfrac{\cancel{x-1}}{\cancel{(x-1)}(x-2)} = \dfrac{1}{1-2} = \dfrac{1}{-1} = \boxed{-1}$

N39 $2x-1 \le f(x) \le x^2$, for $0 \le x < 3$.

we need to find $\lim_{x \to 1} f(x)$

Solution:
Let's use The Squeeze Theorem:

$$\left.\begin{aligned} &\lim_{x \to 1} 2x-1 = 2-1 = 1 \\ &\lim_{x \to 1} x^2 = 1 \end{aligned}\right\} \text{Hence, } \boxed{\lim_{x \to 1} f(x) = 1}$$

N40 Prove that $\lim_{x \to 0} x^2 \cos\left(\frac{1}{x^2}\right) = 0$

Proof: Let's use the squeeze theorem again:

$-1 \le \cos\frac{1}{x^2} \le 1$, hence

$-x^2 \le x^2 \cos\frac{1}{x^2} \le x^2$

$\lim_{x \to 0} (-x^2) = 0$ and $\lim_{x \to 0} x^2 = 0$, therefore

$\lim_{x \to 0} x^2 \cos\left(\frac{1}{x^2}\right) = 0$

q.e.d.

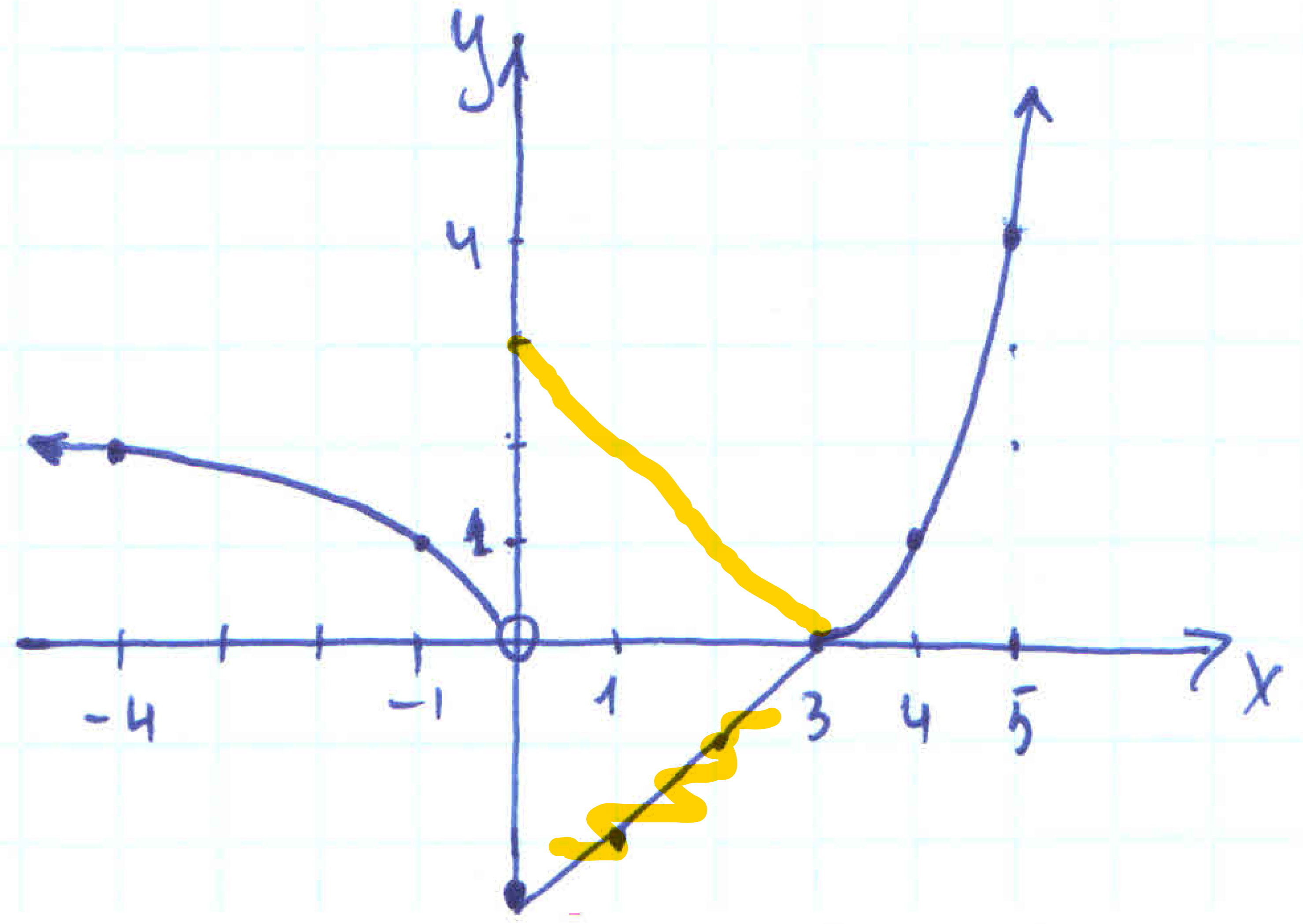
N45

a) (i) $\boxed{3}$ (ii) $\boxed{0}$ (iii) $\boxed{\text{does not exist}}$

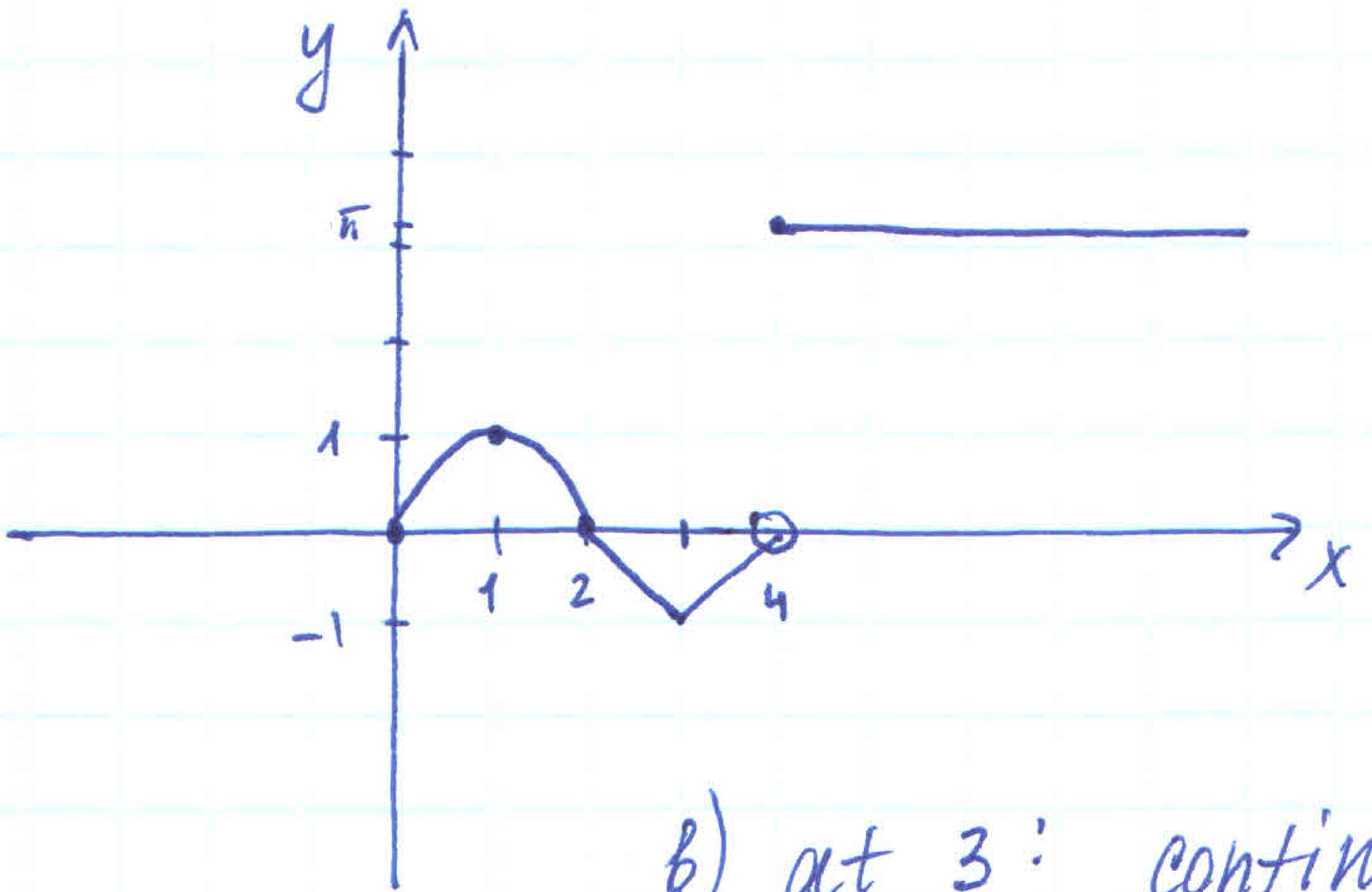
(iv) $\boxed{0}$ (v) $\boxed{0}$ (vi) $\boxed{0}$

b) at $\boxed{0}$

c)



N46 let's do (b) first.



(a) at 2: continuous, because

$$\left.\begin{array}{l}\lim\limits_{x\to 2^-} g(x) = 0 \\ \lim\limits_{x\to 2^+} g(x) = 0\end{array}\right\} \text{hence } \lim\limits_{x\to 2} g(x) = 0,$$

and $g(2) = 2\cdot 2 - 2^2 = 4 - 4 = 0$

b) at 3: continuous. (same way of reasoning)

c) at 4: continuous from the right (only), because

$$\lim_{x\to 4^+} g(x) = \pi = g(4)$$

N47 $h(x) = \sqrt[4]{x} + x^3 \cos x$

Solution: domain: $\{x \in \mathbb{R} \mid x \geq 0\}$ - all non-negative real numbers

at any x from the domain,

$\sqrt[4]{x}$ is defined, and $x^3 \cos x$ is defined on x

root function is continuous on its domain (Theorem 9)

polynomial function is continuous everywhere

trigon. function is continuous on its domain ($\mathbb{R}$)

Hence, by Theorem 4. 1), 3), 4) prop-ties, $h(x)$ is continuous on its domain: $\{x \in \mathbb{R} \mid x \geq 0\}$

q.e.d.

N48 $g(x) = \dfrac{\sqrt{x^2-9}}{x^2-2}$

Solution: domain: $\{x \in \mathbb{R} \mid x \neq \pm\sqrt{2},$ ~~because~~ $x \geq 3$ or $x \leq -3\}$

similar to the previous proof: $\sqrt{x^2-9}$ ← root function, continuous on $\{x \in \mathbb{R} \mid x \geq 3$ or $x \leq -3\}$; x^2-2 - polynomial function, continuous on $\mathbb{R}$

Then by Theorem 4 5) prop-ty, $g(x)$ is continuous on $\{x \in \mathbb{R} \mid x \neq \pm\sqrt{2},\ x \geq 3 \text{ or } x \leq -3\}$.

q.e.d.

$\boxed{N49}$ $x^5 - x^3 + 3x - 5 = 0$ $(1,2)$

Solution: let $f(x) = x^5 - x^3 + 3x - 5$

then $f(1) = 1 - 1 + 3 - 5 = -2$, and
$f(2) = 32 - 8 + 6 - 5 = 25$.

Thus, by Intermediate Value Theorem, there is a value $a \in (1,2)$ such that $f(a) = 0$.

q.e.d.

$\boxed{N50}$ $2\sin x = 3 - 2x$, $(0,1)$

Solution: let $f(x) = 2\sin x - 3 + 2x$

then $f(0) = 0 - 3 + 0 = -3$, and
$f(1) \approx 0.841$

Hence, by Intermediate Value Theorem, there is a value $a \in (0,1)$ such that $f(a) = 0$

q.e.d.